Constraints on Compact Hyperbolic Spaces from COBE

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The (large angle) COBE DMR data can be used to probe the global topology of our universe on scales comparable to and just beyond the present "horizon". For compact topologies, the two main effects on the CMB are: [1] the breaking of statistical isotropy in characteristic patterns determined by the photon geodesic structure of the manifold and [2] an infrared cutoff in the power spectrum of perturbations imposed by the finite spatial extent. To make a detailed confrontation of these effects with the COBE maps requires the computation of the pixel-pixel temperature correlation function for each topology and for each orientation of it relative to the sky. We present a general technique using the method of images for doing this in compact hyperbolic (CH) topologies which does not require spatial eigenmode decomposition. We demonstrate that strong constraints on compactness follow from [2] and that these limits can be improved by exploiting the details of the geodesic structure for each individual topology ([1]), as we show for the flat 3-torus and selected CH models.

Flat or open FRW models adequately describe the observed average properties of our Universe. Much recent astrophysical data suggest the cosmological density parameter Ω_0 is < 1. [1] In the absence of a cosmological constant, this would imply a hyperbolic 3-geometry for the universe. There are numerous theoretical reasons, however, to favour compact topologies (reviewed in ref. 2). To reconcile this with a flat or hyperbolic geometry, compact models can be constructed by identifying points on the standard infinite FRW space by the action of certain allowed discrete subgroups of isometries, Γ. The FRW spatial hypersurface is then the "universal cover", tiled by copies of the compact space.[a] Dynamical chaos arising from the resulting complex geodesic structure in CH spaces has been proposed as an explanation of the observed homogeneity of the universe.[3]

Any unperturbed FRW spacetime will have an isotropic cosmic microwave background (CMB) regardless of global topological structure. However, the topology does affect the observed CMB temperature fluctuations $\Delta T/T$ and thus can be tested. At large angular scales, $\Delta T/T$ is dominated by the Sachs-Wolfe effect: $\Delta T/T \propto \Phi$, where Φ is the gravitational potential, appropriately smoothed to take into account the COBE beam. It is usually computed

[a] Any point $\mathbf{x}$ of the compact space has an image $\mathbf{x}_i = g_i\mathbf{x}$ in each "Dirichlet" domain on the universal cover, where $g_i \in \Gamma$. Compact hyperbolic manifolds (CHM) are described by discrete subgroups of the proper Lorentz group. A census of CHMs and software (SnapPea) for computing the generators of Γ for any CHM is freely available.[4] CHMs can be classified in terms of V/d_c^3, where V is the volume and $d_c = H_0^{-1}/\sqrt{1-\Omega_0}$ is the curvature radius.[5]

by decomposing into eigenmodes of the 3-Laplacian for the space, *e.g.* just plane waves for flat universes. We avoid the nontrivial task of finding these eigenmodes for a CH space by using the fact that the correlation function $C_c \equiv \langle \Phi(\mathbf{x})\Phi(\mathbf{x}')\rangle$ in a compact space can be expressed as a sum over the correlation function C_u in its universal covering space between the images of $\mathbf{x}$ and $\mathbf{x}'$:

$$C_c(\mathbf{x},\mathbf{x}') = \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N} \sum_{j=0}^{N} C_u(g_i\mathbf{x}, g_j\mathbf{x}')\,. \tag{1}$$

The g_i are ordered in increasing displacement and g_0 is the identity. This procedure can be applied to any compact space provided the set of elements $\{g_i\}$ of the symmetry group is known. For a flat gravitational potential perturbation spectrum, $C_u(\mathbf{x},\mathbf{x}') \propto \int d\ln(\beta^2+1)\,[\sin(\beta \mathrm{r})/(\beta \sinh r)]$, where r is the proper separation between $\mathbf{x}$ and $\mathbf{x}'$.

The prescription (1) applies separately to each "spectral mode", $C_c(\beta,\mathbf{x},\mathbf{x}')$, defined as the integrand in $C_c(\mathbf{x},\mathbf{x}') \propto \int d\ln(\beta^2+1) C_c(\beta,\mathbf{x},\mathbf{x}')$. This decomposition is useful since the contribution to the $\ell^{\rm th}$ multipole in the CMB angular correlation function comes predominantly from scales $\beta_\ell \approx \ell d_c/\mathcal{R}_H$ where $\mathcal{R}_H$ is the "angle-diameter distance" to the last scattering surface. In all the CH models that we have studied, compactness leads to a cutoff in $C_c(\beta,\mathbf{x},\mathbf{x}')$ at scales at least four times the circum-radius of the space, $R_>$, *i.e.*, for $\beta < \beta_{cut} \approx (\pi/2)R_>^{-1}$ (see Fig. 1). Thus the $\ell^{\rm th}$ multipole will be strongly suppressed if $\beta_{cut} > \beta_\ell$. We consider a model to contradict the COBE data if the $\ell \leq 4$ multipoles are strongly suppressed, translating to a criterion that the two parameters of the problem, $R_>/d_c$ and $\mathcal{R}_H/d_c$, a function only of Ω_0, have to satisfy: $R_> > (\pi/8)\mathcal{R}_H$. Fig. 1 shows our constraints in the $\Omega_0 - R_>$ parameter plane. We have studied some representative CH models (dots in Fig.1) in detail by confronting the full predicted statistics of the CMB anisotropy pattern with the all-channel COBE map.[6] We find that the exact likelihood falls steeply once the cutoff wavelength is reduced to a size comparable to the horizon, and the disallowed region in Fig. 1 determined by the simple β_{cut} criterion is strongly ruled out.

We conclude from Fig. 1 that the bounding scale $R_>$ of the universe cannot be much smaller than $\mathcal{R}_H$. This makes problematical topological explanations of galaxy and quasar distributions,[2,8] but there may be cases for which $R_>$ is large yet some closed geodesics are much shorter. Full statistical analysis using the COBE maps tightens the limits in all spaces considered so far.[6] When we apply our full statistical method to flat (equal-sided) 3-tori, we improve upon previous limits [7] on the size of the torus, $d_T \gtrsim 4H_0^{-1}$ (95% CL), or $R_> \gtrsim \sqrt{3}\mathcal{R}_H$, much stronger than the conservative β_{cut} criterion given above because

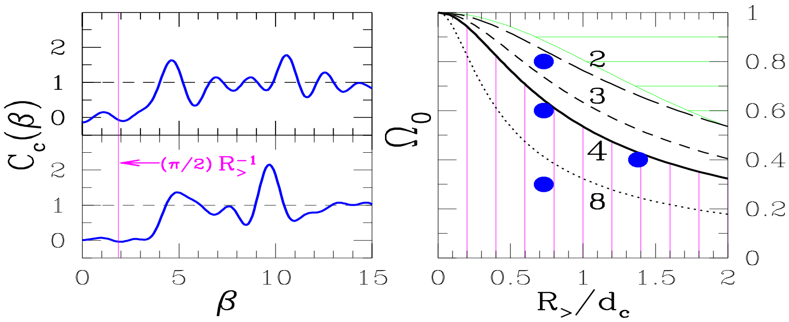

Figure 1: **left panel:** Examples of $C_c(\beta, \mathbf{x}, \mathbf{x}')$ for zero pixel separation in a CH model with $R_> = 0.83d_c$. Normalization is such that $C_u(\beta) = 1$. The infrared cutoff is present for $\mathbf{x} \neq \mathbf{x}'$ as well. **right panel:** Ω_0–$R_>$ constraints on CH models. Models in the region below the labeled lines have no power at multipoles $\ell \leq 2, 3, 4, 8$, respectively. In the upper shaded region, the compact space's volume exceeds the volume of a sphere of radius $\mathcal{R}_H$. Models in the lower shaded region fail our cut criterion with ℓ=4. The large dots denote some examples for which a full statistical comparison with the DMR data has shown the topologies to be incompatible. This indicates the breaking of isotropy can lead to models outside of the lower shaded region being ruled out.

of the powerful breaking of statistical isotropy in the torus case. For compact hyperbolic universes, the low values of Ω_0 suggested by various astrophysical observations, $\lesssim 0.4$, are excluded for all the known [4] spaces. However, for $\Omega_0 \approx 1$, the multifaceted topology makes CH models more compatible with the COBE data than flat $\Omega_0 \equiv 1$ torus models.

References

1. Dekel, A., Burnstein, D., & White, S.D.M., 1996, in *Critical Dialogues in Cosmology*, ed. N.Turok, World Scientific.
2. Lachieze-Rey, M. & Luminet, J.-P. 1995, Phys. Rep. **25**, 136
3. Cornish, N.J., Spergel, D.N. & Starkman, G.D. 1996, Phys.Rev.Lett. **77**, 215
4. Weeks, J.R., *SnapPea: A computer program for creating and studying hyperbolic 3-manifolds*, University of Minnesota Geometry Center.
5. Thurston, W.P. 1979, *The Geometry of 3-Manifolds*, lecture notes, Princeton University; Thurston, W.P. & Weeks, J.R. 1984, Sci. Am. (July), 108
6. Bond, J.R., Pogosyan, D. & Souradeep, T. 1997, preprint
7. Sokolov, I.Y. 1993, JETP Lett. **57**, 617; Starobinsky, A.A. 1993, JETP Lett. **57**, 622; Stevens, D., Scott, D. & Silk, J. 1993, Phys. Rev. Lett. **71**, 20; de Oliveira Costa, A. & Smoot, G.F. 1995, Ap.J. **448**, 477; de Oliveira Costa, A., Smoot, G.F. & Starobinsky, A.A. 1996, Ap.J. **468**, 457
8. *e.g.*, Fagundes, H.V. 1989, Ap.J. **338**, 618; Roukema, B.F. 1996, preprint